U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

FIRST YEAR

ELEMENTS OF CALCULUS

Time: 3 Hours **Maximum Marks: 75**

PART – A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. Find y_n, when $y = \frac{x^2}{(x+1)^2(x+2)}$.

2. If $z = f(x,y)$ and $x = rcos\theta, y = rsin\theta$ prove that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$

3. Find the envelope of the family of straight lines $y + tx = 2at + at^2$, where t is the parameter.

4. Find the value of $\int_0^a \int_0^x (x^2 + y^2) dydx$

5. Obtain reduction for $\int_o^n tan^n x dx; n \in N$

6. Prove that $\lim_{n\to\infty} \frac{x^n}{n!} = 0$

7. Discuss the convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \cdots$

8. Test the convergence of $\sum \frac{2^n n!}{n^n}$

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. If $y = \mathrm{a}\cos(logx) + bsin(logx)$ prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

10. Investigate the maximum and minimum values of $2(x^2 - y^2) - x^4 + y^4$.

11. Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

12. Find the reduction formula for $I_n = \int x^m (logx)^n dx; m, n \in N$ and using this find the value of $\int x^4 (logx)^3 dx$

13. Find the volume of the cylinder $x^2 + y^2 = a^2$ above the xy-plane cut by the plane $x + y + z = a$.

14. Show that the sequence $\left(1 + \frac{1}{n}\right)^n$ converges.

15. Discuss the convergence of the series $\sum \frac{\sqrt{n+1-\sqrt{n}}}{n^p}$

16. Test the convergence of $\sum \frac{n^3+a}{2^n+a}$

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

FIRST YEAR

ELEMENTS OF CALCULUS

Time: 3 Hours **Maximum Marks: 70**

PART - A **(5 x 2 = 10 Marks)**

Answer all FIVE questions.

1. State Euler's theorem (without proof).

2. Find the envelop of the family of a straight lines $y + tx = 2at + at^3$, the parameter being t.

3. Define the beta function and gamma function.

4. Prove that $lim_{n\to\infty} \frac{1}{2^n} = 0$.

5. Test the convergence of the series $\sum \frac{1}{(logn\,)^n}$ by root test.

PART - B **(4 x 5 = 20 Marks)**

Answer any FOUR questions.

6. Find the n^{th} differential coefficient of $cos^5\theta \cdot sin^7\theta$.

7. Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

8. Find the area of the surface of the sphere of radius r.

9. Show that the sequence $\left(1 + \frac{1}{n}\right)^n$ converges.

10. Prove that the series $\sum \frac{1}{n^p}$ converges if $p > 1$ and divergies if $p \leq 1$.

11. State and prove Cauchy's second limit theorem.

12. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ where $m, n > 0$.

PART - C **(4 x 10 = 40 Marks)**

Answer any FOUR questions.

13. Discuss the maxima and minima of the function $x^3y^2(6-x-y)$.

14. (i) Prove that the radius of curvature at any point of the cycloid $x = a(\theta + sin\theta)$ and $y = a(1 - cos\theta)$ is $4acos\frac{\theta}{2}$
(ii) Show that the chord of curvature through the focus of a parabola is four times the focal distance of the point and the chord of curvature parallel to the axis has the same length.

15. A plane lamina of non-uniform density is in the form of a quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If the density at any point (x, y) be Kxy, where K is a constant, then find the co-ordinates of the centroid of the lamina.

16. If the sequences (a_n) and (b_n) converge to 0 and (b_n) is strictly monotonic decreasing then prove that $lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = lim_{n\to\infty}\left(\frac{a_n - a_{n+1}}{b_n - b_{n+1}}\right)$provided the limit on the right hand side exists whether finite or infinite.

17. State and prove comparison test in detail. Use comparison test to prove that the series $\sum\frac{1^2+2^2+\cdots+n^2}{n^4+1}$ is a divergent series.

18. If $(a_n) \to a$ and $a_n > 0$, $a > 0$ for all n then prove that (i) $\left(\frac{1}{a^n}\right) \to \frac{1}{a}$ and
(ii) $\sqrt{a_n} \to \sqrt{a}$

19. (i) Evaluate $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.
(ii) Change the order of integration in the integral $\int_0^a \int_{x^2/a}^{2a-x} xy\,dx\,dy$ and evaluate it.

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

FIRST YEAR

TRIGNOMETRY, ANALYTICAL GEOMETRY (3D) AND VECTOR CALCULUS

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. If $\tan(a+ib) = x+iy$, prove that $\frac{x}{y} = \frac{sin\,2a}{sin\,h2b}$.

2. If $i^{a+ib} = a+ib$, prove that $a^2+b^2 = e^{-(4n+1)\pi b}$.

3. Find the equation of plane passing through (2,2,1) and (9,3,6) and perpendicular to the plane $2x+6y+6z=9$.

4. Show that the lines $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and $\frac{x-5}{2} = \frac{y-8}{3} = \frac{z-7}{2}$ are coplanar and find the equation of the plane containing them.

5. Find the equation of the sphere passing through the points (0,0,0), (1,0,0), (0,1,0) and (0,0,1).

6. Show that $div\left(\frac{\vec{r}}{r}\right) = \frac{2}{r}$.

7. Obtain the directional derivative of $\varphi = xy^2 + yz^3$ at the point $(2,-1,1)$ in the direction of $\vec{\imath} + 2\vec{\jmath} + 2\vec{k}$.

8. Using Green's theorem evaluate $\int_c (xy - x^2)dx + x^2 y dy$ along the closed curve C formed by $y = 0, x = 1$ and $y = x$.

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. Prove that $cos8\theta = 128cos^8\theta - 256cos^6\theta + 160cos^4\theta - 32cos^2\theta + 1$

10. Sum to infinity the series $cos\alpha + \frac{1}{2}cos(\alpha+\beta) + \frac{1.3}{2.4}\cos(\alpha+2\beta) + \cdots\infty$

11. Find the shortest distance between the lines
$2x-2y+3z-12 = 0 = 2x+2y+z$ and $2x-z = 0 = 5x-2y+9$.

12. Find the image of the point (1,3,4) under the reflection in the plane $2x - y + z + 3 = 0$. Hence, prove that the image of the line $\frac{x-1}{1} = \frac{y-3}{-2} = \frac{z-4}{-3}$ is $\frac{x+3}{1} = \frac{y-5}{-5} = \frac{z-2}{-10}$.

13. Show that the plane $2x - 2y + z + 12 = 0$ touched the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$. Also, find the point of contact.

14. Prove: $\nabla \times \left(\nabla \times \vec{F}\right) = \nabla\left(\nabla \cdot \vec{F}\right) - \nabla^2 \vec{F}$.

15. Evaluate $\int \int_S \vec{f} \cdot \vec{n} dS$, where $\vec{f} = (x + y^2)\vec{i} - 2x\vec{j} + 2yz\vec{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

16. Verify Gauss divergence theorem for $\vec{f} = y\vec{i} + x\vec{j} + z^2\vec{k}$ for the cylindrical region S given by $x^2 + y^2 = a^2; z = 0$ and $z = h$.

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U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

FIRST YEAR

TRIGNOMETRY, ANALYTICAL GEOMETRY (3d) AND VECTOR CALCULUS

Time: 3 Hours **Maximum Marks: 70**

PART - A **(5 x 2 = 10 Marks)**

Answer all **FIVE** questions in 50 words
[All questions carry equal marks]

1. Write the expansion of $\tan\theta$ (without proof).

2. Find the equation of the plane passing through (1,1,0), (1,2,1) and $(-2, 2, -1)$.

3. Obtain the equation of the sphere having its centre at the point (6, -1, 2) and touching the plane $2x - y + 2z - 2 = 0$.

4. If $\boldsymbol{u}(t) = x(t)\boldsymbol{i} + y(t)\boldsymbol{j} + z(t)\boldsymbol{k}$ and $\boldsymbol{v}(t) = X(t)\boldsymbol{i} + Y(t)\boldsymbol{j} + Z(t)\boldsymbol{k}$ then prove that
$$\frac{d}{dt}(\boldsymbol{u}\cdot\boldsymbol{v}) = \boldsymbol{u}\cdot\frac{dv}{dt} + \frac{du}{dt}\cdot\boldsymbol{v}.$$

5. Write the statement of Stokes theorem (without proof).

PART - B **(4 x 5 = 20 Marks)**

Answer any **FOUR** questions out of Seven questions in 150 words
[All questions carry equal marks]

6. Expand $sin^3\theta \cdot cos^5\theta$ in a series of sines of multiples of θ.

7. Prove that the plane $x + 35y - 10z - 156 = 0$ is a bisector of the angle between two planes one of which is $4x - 3y + 12z + 13 - 0$. Also find the equation of the other plane.

8. Find the equations of the tangent planes of the sphere $x^2 + y^2 + z^2 - 4x - 4y - 4z + 10 = 0$ which are parallel to the plane $x - z = 0$.

9. Find the angle between the surfaces $x^2 + y^2 + z^2 = 29$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z - 47 = 0$ at (4, -3, 2).

10. Verify the Gauss divergence theorem for the function $\boldsymbol{f} = a(x + y)\boldsymbol{i} + a(y - x)\boldsymbol{j} + z^2\boldsymbol{k}$ over the hemisphere bounded by the *xoy* plane and the upper half of the sphere $x^2 + y^2 + z^2 = a^2$.

11. Separate into real and imaginary parts of $tan^{-1}(x + iy)$.

12. Find the shortest distance and the equation of the line of shortest distance between the straight lines $\frac{x+3}{-4}=\frac{y-6}{6}=\frac{z}{2}$ and $\frac{x+2}{-4}=\frac{y}{1}=\frac{z-7}{1}$.

PART - C **(4 x 10 = 40 Marks)**

Answer any **FOUR** questions out of Seven questions in 400 words
[All questions carry equal marks]

13. Expand $\cos 9\theta$ in powers of $\cos\theta$ and hence deduce that$cos\frac{\pi}{9}\cdot cos\frac{2\pi}{9}\cdot cos\frac{4\pi}{9}=\frac{1}{8}$.

14. Find the image of the point (1, 3, 4) under the reflection in the plane $2x-y+z=-3$. Hence prove that the image of the line $\frac{x-1}{1}=\frac{y-3}{-2}=\frac{z-4}{-3}$ is $\frac{x+3}{1}=\frac{y-5}{-5}=\frac{z-2}{-10}$.

15. (i) Find the equation of the sphere passing through the circle $x^2+y^2+z^2-4=0, 2x+4y+6z-1=0$ and having its centre on the plane $x+y+z=6$.
(ii) Find the equations of the spheres which passes through the circle $x^2+y^2+z^2-2x+2y+4z-3=0; 2x+y+z-4=0$ and touches the plane $3x+4y-14=0$.

16. Prove that (i) $\boldsymbol{curl(f+g)}=\boldsymbol{curl(f)}+\boldsymbol{curl(g)}$; (ii) grad $(\boldsymbol{f}\cdot\boldsymbol{g})=\boldsymbol{f}\cdot\boldsymbol{curl(g)}+\boldsymbol{g}\ \boldsymbol{X}\ \boldsymbol{curl(f)}+\ (\boldsymbol{f}\cdot\nabla)\boldsymbol{g}+(\boldsymbol{g}\cdot\nabla)\boldsymbol{f}$

17. Verify Stokes theorem for $f=(2x-y)i-yz^2j-y^2zk$ where S in the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary.

18. Prove that the condition for two lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ to be coplanar is $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$. Use this result to prove the lines $\frac{x-2}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ and $\frac{x-5}{2}=\frac{y-8}{3}=\frac{z-7}{2}$ are coplanar and find the equation of the plane containing them.

19. (i) Evaluate $\int_c f\cdot dr$ where $f=(x^2+y^2)i-2xy\,j$ and the curve C is the rectangle in the $x-y$ plane bounded by $y=0$, $y=b$, $x=0$, $x=a$.
(ii) Evaluate $\iint f\cdot n\,dS$ where $f=(x^3-yz)i-2x^2yj+2k$ and S is the surface of the cube bounded by $x=0, y=0, z=0, x=a, y=a$ and $z=a$.

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

FIRST YEAR

DIFFERENTIAL EQUATIONS

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. Solve: $xyp^2 + (x + y)p + 1 = 0$. (Where $p = \frac{dy}{dx}$)

2. Solve: $(px - y)(py + x) = 2p$.

3. Solve: $(D^2 + 4D + 3)y = e^{-3x}$.

4. Solve: $x^2y'' - 3xy' = x + 1$.

5. Solve: $yzlogzdx - zxlogzdy + xydz = 0$

6. Solve: $p^2 + q^2 = x + y$

7. Find Laplace transform of sin^2tcos^3t.

8. Find inverse Laplace transform of $\frac{s-1}{2s^2+s+6}$

PART – B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. Solve: $yp^2 - xp + 2y = 0$

10. Solve: $(D^2 - D + 1)y = x^3 - 3x^2 + 1$

11. Solve: $y'' + y = cosecx$ using variation of parameter method.

12. Solve: $yzdx + (xz - yz^3)dy - 2xydz = 0$

13. Solve: $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

14. Solve: $\left(p^2 + q^2\right)y = qz(p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}\right)$ by Charpit's method.

15. (i) Find $L\left(\frac{\cos 3t-\cos 2t}{t}\right)$

(ii) Find $L^{-1}\left(\frac{S+2}{(S-4)(S^2+1)}\right)$

16. Using Laplace transform solve the equation
$y'' + 6y' + 5y = 2e^{-2t}$; given $y(0) = 0, y'(0) = 1$.

U.G. DEGREE EXAMINATION - JUNE 2021
MATHEMATICS
FIRST YEAR

DIFFERENTIAL EQUATIONS

Time: 3 Hours **Maximum Marks: 70**

PART - A **(5 x 2 = 10 Marks)**

Answer all **FIVE** questions

1. Form the differential equation for which $xy = ae^{x} + be^{-x} + x^{2}$ is a solution.
2. Solve: $y'' + 4y' + 13y = 2e^{-x}$ given $y(0) = 0$ and $y'(0) = -1$.
3. Solve: $\frac{xdx}{y^2z} = \frac{dy}{xz} = \frac{dz}{y^2}$
4. Solve: $x\frac{\partial z}{\partial x} = 2x + y + 3z$
5. Write sufficient conditions for the existence of the Laplace transformation.

PART - B **(4 x 5 = 20 Marks)**

Answer any **FOUR** questions

6. Find the solution of the Clairauts' equation $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$.
7. (i) Solve the exact differential equation $(a^2 - 2xy - y^2)dx - (x+y)^2dy = 0$.
   (ii) Solve: $xy' + y = y^2\log x$.
8. Solve $\frac{dx}{y^2(x-y)} = \frac{dy}{-x^2(x-y)} = \frac{dz}{z(x^2+y^2)}$.
9. Solve $(y + z)p + (z + x)q = x + y$ by Lagrange's method.
10. (i) Find $L\left(\frac{\cos 3t - \cos 2t}{t}\right)$
    (ii) Find $L^{-1}\left(\frac{S+2}{(S-4)(S^2+1)}\right)$
11. Using Laplace transform, solve the differential equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$ given that $y(0) = 0$ and $\frac{dy}{dt}\mathrm{I}_{t=0} = 0$.
12. Solve the differential equation $xy'' - (2x+1)y' + (x+1)y = x^2e^x$.

PART - C **(4 x 10 = 40 Marks)**

Answer any **FOUR** questions

13. Solve: $(x^3 - 3xy^2)dx - (y^3 - 3x^2y)dy = 0$.

14. Apply the method of variation of parameters to solve $x^2y'' + 4xy' + 2y = e^x$.

15. Solve (i) $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$; (ii) $\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}$.

16. Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$.

17. Using Laplace transform techniques, solve the simultaneous equations: $3\frac{dx}{dt} + \frac{dy}{dt} + 2x = 1$; $\frac{dx}{dt} + 4\frac{dy}{dt} + 3y = 0$, given that $x(0) = 0 = y(0)$.

18. (i) Find the Laplace transform of $\frac{\cos 2t - \cos 3t}{t}$, (ii) Find $L^{-1}\left[\frac{s}{s^2a^2+b^2}\right]$

19. Show that the solution of the differential equation $\frac{d^2y}{dt^2} + 4y = Asinpt$, which is such that $y(0) = 0$ and $\frac{dy}{dt}\text{I}_{t=0} = 0$ is $y = \frac{A}{4-p^2}(\sin pt - \frac{1}{2}psin2t)$ if $p \neq 2$. If $p = 2$ then show that $y = \frac{A}{8}(\sin 2t - 2t\cos 2t)$.

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

SECOND YEAR

GROUPS AND RINGS

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any FIVE questions.

1. What is an equivalence relation? Show that if ρ and σ are equivalence relations defined on a set S. Prove that $\rho \cap \sigma$ is an equivalence relation.

2. Show that cube roots of unity with usual multiplication forms a group.

3. Define a Normal subgroup of a group and give an example.

4. Show that every cyclic group is abelian.

5. Let G be a group and let a be a fixed element of G. Let $H_a = \{x/x \in G \ \ and \ ax = xa\}$. Show that H_a is a subgroup of G.

6. Prove that the intersection of two subrings of a ring R is a subring of R.

7. If $f: R \to R^{'}$ is a Ring homomorphism and K is the kernel of f, Prove that K is an ideal of R.

8. Prove that any Field is an Euclidean domain if $d(a) = 1$ for all $a \in F - \{0\}$.

PART - B **(5 x 10 = 50 Marks)**

Answer any FIVE questions.

9. Let G deonote the set of all matrices of the form $\begin{pmatrix} x & x \\ x & x \end{pmatrix}$ where $x \in R^*$ (non-zero real numbers). Prove that G forms a group under matrix multiplication.

10. State and prove Fundamental theorem of homomorphism.

11. Prove that every integral domain can be imbedded in a field.

12. State and prove Cayley's theorem.

13. State and Prove Lagrange's theorem.

14. Let R be a commutative ring with identity. Prove that an ideal M of R is maximal if and only if R/M is a field.

15. (i) Prove that every subgroup of an abelian group is normal.
(ii) Prove that isomorphism is an equivalence relation among groups.

16. Let 'a' be a non-zero element of an Euclidean domain R. Show that a is a unit in R if and m/y if $d(a) = d(I)$.

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

SECOND YEAR

STATISTICS AND MECHANICS

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. Find the mean deviation about the median for the given data 13,17,16,14,11,13,10,16,11,18,12,17.

2. Urn I has two white and three black balls. Urn II has four white and one black balls and Urn III has three white and four black balls. An urn is selected at random and a ball drawn at random is found to be white. Find the probability that Urn I was selected.

3. Fit a line of best of the form $y = ax + b$ for the following data:

Year (x)	1951	1952	1953	1954	1955	1956	1957
Production (y)	201	263	314	395	427	504	612

4. State and Prove the additive property of Moment generating function.

5. Define Type I and Type II error.

6. Find by interpolation the number for 1980 from the following table of index numbers of production of a certain article in India:

Year	1978	1979	1981	1982
Index Number	101	107	102	99

7. Define Poisson distribution and find its mean and variance.

8. Define the following: (i) a Projectile (ii) Period, frequency of SHM

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. Calculate the mean and S.D of the following frequency distribution of marks:

Marks (x)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of Students (f)	5	2	30	45	50	37	21

10. State and Prove Chebychev's inequality.

11. Determine the Karl Pearson's coefficient of correlation between X and Y.

X	10	12	13	12	16	15
Y	40	38	43	45	37	43

12. In a certain sample of 2000 families 1400 families are consumers of tea. Out of 1800 Hindu families, 1236 families consume tea. Use X^2 test and State whether there is any significant difference between consumption of tea among Hindu and Non-Hindu families.

13. Two random samples gave the following results.

Sample	Size	Sample mean	Sun of squares of deviations from the mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population.

14. Construct the consumer price index number for the year 1984 on the basis of the following data using the average expenditure method and Laspeyre's method.

Commodities	Quantity consumed in 1986	Unit	Price in 1985 in rupees	Price in 1986 in rupees
A	6 Quintal	Quintal	5.75	6
B	6 Quintal	Quintal	5	8
C	1 Quintal	Quintal	6	9
D	6 Quintal	Quintal	8	10
E	4 Kg	Kg	2	1
F	1 Quintal	Quintal	20	15

15. A particle is executing a simple harmonic motion with O as the mean position and a is the amplitude. When it is at a distance $\frac{a}{2}$ from O its velocity is quadrupled by a blow. Show that its new amplitude is $7\frac{a}{2}$.

16. State and prove the equation of the central orbit in pedal form.

UG-A-445	BMS-23

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

SECOND YEAR

CLASSICAL ALGEBRA AND NUMERICAL METHOD

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$. Given that one of the roots is $1 - \sqrt{5}$.

2. Find the equation each of whose roots exceeds by 2, root of the equation $x^3 - 4x^2 + 3x - 1 = 0$.

3. Find the sum of the series to infinity $\log 2 - \frac{(log\, 2)^2}{2!} + \frac{(log\, 2)^3}{3!} + \cdots$

4. Find the sum of the series $\frac{1}{1\cdot 2} + \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} + \cdots \infty$

5. Find the approximate real root of the equation $xe^x - 3 = 0$. $1 < x < 1.1$ using Regula Falsi method.

6. Using Newton divided difference formula find the missing value using the table

x	1	2	4	5	6
y	14	15	5	?	9

7. Find $f'(4)$ from the table

x	0	2	3	5
f(x)	8	6	20	108

8. Using Euler's method find $y(0.2)$ given that $y' = x + y$ given that $y(0) = 1$

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. Find the sum to infinity of the series $\frac{7}{72}+\frac{7.28}{72.96}+\frac{7.28.49}{72.96.120}+\cdots$.

10. Sum to infinity the series $\frac{2.3}{3!}+\frac{3.5}{4!}+\frac{4.7}{5!}+\frac{5.9}{6!}+\cdots$

11. Solve: $6x^5+11x^4-33x^3-33x^2+11x+6=0$

12. If α,β,γ are the roots of equation $x^3+px^2+qx+r=0$ find the value of
    (i) $\sum\alpha^2$
    (ii) $\sum\frac{1}{\alpha}$
    (iii) $\sum\frac{1}{\alpha\beta}$
    (iv) $\sum\alpha^2\beta^2$

13. Apply Gauss Jordan method to solve the system
$x+y+z=9$
$2x-3y+4z=13$
$3x+4y+5z=40$

14. Using Lagrange's formula fit a polynomial to the data:

x	-1	0	2	3
y	-8	3	1	12

and hence find y when $x=1$

15. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ using Trapezoidal rule by taking h=0.25 and by taking h= 0.5.

16. Apply the fourth order Runge-Kutta method, to find an approximate value of y when $x=0.2$ given that $\frac{dy}{dx}=x+y, y(0)=1$.

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

THIRD YEAR

REAL AND COMPLEX ANALYSIS

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. Prove that $[0,1] = \{x/0 \leq x \leq 1\}$ is uncountable.

2. Prove if f is a continuous function from a metric space M_1 into a metric spaceM_2 and if M_1 is connected, then the range of f is also connected.

3. Find the Taylor series about $x = 2$ for
$$f(x) = x^3 + 2x + 1 \qquad (-\infty < x < \infty)$$

4. Prove that if A is a closed subset of the compact metric space $\langle M, \rho \rangle$ then metric space $\langle A, \rho \rangle$ is also compact.

5. Show that $f(z) = sinz$ is an analytic function.

6. Find the image of $|z - 2i| = 2$ under the transformation $w = \frac{1}{z}$.

7. Find the invariant points of the transformation $w = \frac{2z+4i}{1+iz}$

8. Expand $f(z) = \frac{1}{z(z-1)}$ as Laurent's series valid in $|z| < 1$ and $< |z| < 2$.

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. Prove that the countable union of countable sets is countable.

10. Prove that if M is a compact metric space, then M has the Heine-Borel property.

11. State and Prove second fundamental theorem of calculus.

12. Prove that a subset of R is connected if and only if it is an interval.

13. Show that an analytic function with (i) constant real part is constant and (ii) constant modulus is constant.

14. State and prove Cauchy residue-theorem.

15. Find the bilinear transformation which maps the points $z_1 = \infty, z_2 = i, z_3 = 0$ on to the points $w_1 = 0, w_2 = i, w_3 = \infty$ respectively.

16. Evaluate $\int_0^{2\pi} \frac{d\theta}{1+acos\theta} (a^2 < 1)$ using contour integration.

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

THIRD YEAR

LINEAR ALGEBRA AND BOOLEAN ALGEBRA

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. Prove that the intersection of two subspaces of a Vector space is a subspace.

2. Prove in $V_3(R)$ the vectors (1,2,1), (2,1,0) and $(1,-1,2)$ are linearly independent.

3. Find the linear transformation $T: V_3(R) \rightarrow V_3(R)$ determined by the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ with respect to the standard basis.

4. Define norm of a vector $x \in V$, where V is an inner product space. Also, Prove $\|\alpha x\| = |\alpha|\|x\|$ for all $\alpha \in F, x \in V$.

5. If S is any subspace of an inner product space V then $S^{\perp}$ is a subspace of V.

6. Let f be the bilinear form defined on $V_2(R)$ by $f(x,y) = x_1y_1 + x_2y_2$ where $x = (x_1, x_2)$ and $y = (y_1, y_2)$. Find the matrix of f with respect to the basis $\{(0,1), (1,0)\}$.

7. Prove that in any distributive lattice if $x \vee a = y \vee a$ and $x \wedge a = y \wedge a$ then $x = y$.

8. Define a Lattice. Give two examples.

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. Let V be a finite dimensional vector space over F and W be a subspace of V, then prove that $dimW \leq dimV$ and $dim\frac{V}{W} = dimV - dimW$

10. Let V and W be two finite dimensional vector spaces over F. Let $dimV = m$ and $dimW = n$. Then Prove that $L(V,W)$ is a vector space of dimension mn over F.

11. Prove that every finite dimensional inner product space has an orthonormal

basis.

12. Let V be a vector space over a field F such that $S, T \subseteq V$ then prove the following:
(i) $S \subseteq T \Rightarrow L(S) \subseteq L(T)$ (ii) $L(S \cup T) = L(S) + L(T)$

13. If V is finite dimensional inner product space and W is a subspace of V then prove that $V = W \oplus W^{\perp}$ (ie., V is the direct sum of W and $W^{\perp}$).

14. Reduce the Quadratic form to diagonal form

$$x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 8x_1x_3$$

15. If B is a Boolean algebra, then Prove
(i) $(a \vee b)' = a' \wedge b'$ (ii) $(a \wedge b)' = a' \vee b'$ (iii) $(a')' = a$

16. If L is a lattice and $a, b, c, d \in L$, then prove the following
    (i) when $a \leq b$ and $c \leq d$ prove $a \vee c \leq b \vee d$ and $a \wedge c \leq b \wedge d$
    (ii) $a \vee a = a$
    (iii) $a \wedge a = a$

U.G. DEGREE EXAMINATION - JUNE 2021

MATHEMATICS

THIRD YEAR

LINEAR PROGRAMMING AND OPERATIONS RESEARCH

Time: 3 Hours **Maximum Marks: 75**

PART – A **(5 X 5 =25 Marks)**

Answer any FIVE of the following.

1. Use simplex method to maximize $Z = 5x_1+4x_2$
   subject to the constraints:
   $4x_1+5x_2\leq 10$, $3x_1+ 2x_2 \leq 9$,
   $8x_1+ 3x_2\leq 12$, $x_1, x_2\leq 0$.

2. Show that the dual of the dual is the primal.

3. Explain Northwest-corner method in transportation problem.

4. Obtain the initial basic feasible solution to the following transposition problem by least cost method.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

5. For a game with the following pay-off matrix, determine the optimal strategies and the value of the game.

$$\begin{array}{cc} & A \\ B & \begin{bmatrix} 6 & -3 \\ -3 & 0 \end{bmatrix} \end{array}$$

6. A and Beach takes out one or two matches and guess how many matches opponent has taken. If one of the players guesses correctly then loser has to pay him as many rupees as the sum of the numbers held by both players. Otherwise, the payout is zero. Write down the pay-off matrix and obtain the optimal

strategies of both the players.

7. A company uses rivets at a rate of 5,000 kg. per year, rivets costing Rs. 2 per kg. It cost Rs. 20 to place an order and the carrying cost of inventory is 10% per year. How frequently should order for rivets be placed and how much?

8. Customers arrive at a box office window, being manned by single individual, according to a Poisson input process with a mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with a mean 90 seconds. Find the average waiting time of a customer.

PART – B **(5 X 10 =50 Marks)**

Answer any FIVE of the following.

9. Use Big-M method to solve the following linear programming problem.

Maximize $z = 2x_1 + x_2 + 3x_3$

subject to the constraints

$x_1 + x_2 + 2x_3 \leq 5$, $2x_1 + 3x_2 + 4x_3 = 12$

and $x_1, x_2, x_3 \geq 0$.

10. Solve the following linear programming problem by dual simplex method.

Minimize $z = x_1 + 2x_2 + 3x_3$

subject to the constraints

$x_1 - x_2 + x_3 \geq 4$, $x_1 + x_2 + 2x_3 \leq 8$

$x_2 - x_3 \geq 2$, and $x_1, x_2, x_3 \geq 0$.

11. Solve the following transportation problem to minimize the total cost, obtaining the initial solution by Vogel's approximation method. If the optimal solution is not unique, find an alternative optimum solution.

					Availabl e
	7	9	3	2	16
	4	4	3	5	14
	6	4	5	8	20
Requirement	11	9	22	8	

12. Five jobs are to be processed and five machines are available. Any machine can process any job with the resulting profit as follows.

Job	**Machine**				
	A	**B**	**C**	**D**	**E**
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

What is the maximum profit that may be expected if an optimum assignment is made?

13. Discuss the EOQ model with constant rate of demand and variable order cycle time where the shortages are allowed.

14. A supermarket has two sales girls running up sales at the counters if the service time for each customer is exponential with mean 4 minutes and if people arrive in a Poisson fashion at the rate of 10 an hour, (i) what is the probability of having to wait for service? (ii) what is the expected percentage of idle time for each girl? (iii) If a customer has to wait, what is the expected length of his waiting time?

15. Customers arrive at a box office window, being manned by single individual, according to a Poisson input process with a mean rate of 30 per hour. The time required to serve a customer has an exponential distribution with a mean of 90 seconds. Find (i) the average waiting time of a customer, (ii) the average number of customers in the system and (iii) the average queue length.

16. Solve the game whose pay-off matrix is given below by graphical method.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{bmatrix} 4 & -2 & 3 & -1 \\ -1 & 2 & 0 & 1 \\ -2 & 1 & -2 & \end{bmatrix} \end{array}$$

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MATHEMATICS

THIRD YEAR

OPTIMIZATION TECHNIQUES

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any **FIVE** of the following.

1. Define slack, surplus and artificial variables.

2. Obtain the dual problem of the following linear programming problem.

Maximize $Z = 2x_1 + x_2 + x_3$
subject to the constraints
$x_1 + x_2 + x_3 \geq 6,$ $3x_1 - 2x_2 + 3x_3 = 3$
$-4x_1 + 3x_2 - 6x_3 = 1$ and $x_1, x_2, x_3 \geq 0$

3. What is an assignment problem? When do you say that it is unbalanced?

4. Obtain the initial basic feasible solution to the following transportation problem by least cost method.

5. Consider the game G with the following pay-off

		Player B	
		B_1	B_2
Player A	A_1	2	6
	A_2	-2	λ

Show that G is strictly determinable for any value of λ and determine the value of the game.

6. Two players A and B match coins. If the coins match, then A wins two units of value. If the coins do not match, then B wins 2 units of value. Determine the optimum strategies for the player and the value of the game.

7. A manufacturer has to supply his customer with 600 units of his product per year. Shortages are not allowed and the storage cost amounts to Re. 0.60 per unit per year. The set-up cost per run is Rs. 80.00. Find the optimum run-size and the minimum average yearly cost.

8. A T.V. repairman finds that the time spent on his jobs has an exponential

distribution with mean 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8-hour day, what is repairman's expected idle time each day? How many jobs ahead of the average set just brought in?

PART - B **(5 x 10 = 50 Marks)**

Answer any **FIVE** of the following.

9. Use simplex method to solve the linear programming problem.

Maximize $z = 2x_1 + 4x_2 + x_3 + x_4$

Subject to the constraints

$x_1 + 3x_2 + x_4 \leq 4,$ $\quad 2x_1 + x_2 \leq 3$

$x_2 + 4x_3 + x_4 \leq 3,$ $\quad x_1, x_2, x_3, x_4 \geq 0$

10. Use dual simplex method to solve the following linear programming problem.

Maximize $z = -3x_1 - 2x_2$

Subject to the constraints

$x_1 + x_2 \geq 1,$ $\quad x_1 + x_2 \leq 7$

$x_2 + 2x_3 \geq 10,$ $\quad x_1 \leq 3$ and $x_1, x_2 \geq 0.$

11. Find the starting solution in the following transportation problem by Vogel's approximation method. Also obtain the optimum solution.

	D_1	D_2	D_3	D_4	Supply
S_1	3	7	6	4	5
S_2	2	4	3	2	2
S_3	4	3	8	5	3
Demand	3	3	2	2	

12. Five jobs are to be processed and five machines are available. Any machine can process any job with the resulting profit as follows.

Job	Machine				
	A	**B**	**C**	**D**	**E**
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

What is the maximum profit that may be expected if an optimum assignment is made?

13. Solve the game whose pay-off matrix is given below by graphical method.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \begin{bmatrix} 4 & -2 & 3 & -1 \\ -1 & 2 & 0 & 1 \\ -2 & 1 & -2 & 0 \end{bmatrix} \end{array}$$

14. Two companies A and B are competing for the same product. Their different strategies are given in the following pay-off matrix:

$$\begin{array}{cc} & \textit{Company A} \\ \text{Company B} & \begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \end{bmatrix} \end{array}$$

Use linear programming to determine the best strategies for both the players.

15. Explain the queuing model (M/M/1):(∞/FCFS), obtain its steady state solution. Also derive the formula for average number of customer in the queue.

16. An item sells for Rs. 4 per unit but 10% discount is offered for lots of 150 units or more. A manufacturing unit that consumes this item at the rate of 20 items per day wants to decide whether or not to take advantage of the discount. The setup cost for ordering a lot is Rs. 50 and the holding cost per unit per day is Re. 0.30. Will it be economical for the manufacturing unit to take advantage of the discount? If yes, then determine the range on the percentage discount in the price of the item that when offered for lots of size 150 or more will not result in any financial advantage to the manufacturing unit.

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MATHEMATICS

THIRD YEAR

PROGRAMMING IN C AND C++

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5 x 5 = 25 Marks)**

Answer any FIVE of the following.

1. List and explain the various data types in C with examples.
2. What is recursion? Explain with an example.
3. Write the difference between union and structure.
4. Write a C++ program to find average of two numbers using friend function.
5. Describe the various looping statements in C++ with examples.
6. Write a program to find sum of N integers using arrays.
7. Explain call by reference with an example.
8. Explain any four library functions with examples.

PART - B **(5 x 10 = 50 Marks)**

Answer any FIVE of the following.

9. What is an expression? Explain the different types of expression in C.
10. Explain about passing arrays to functions with an example C program.
11. Give a brief account on self-referential structure.
12. Explain the various types of inheritances in C++ with examples.
13. Describe about File Handling functions in C with examples.
14. Explain the concept of passing pointers to functions with an example program.
15. Write a program to overload unary operator using friend functions.
16. Discuss the various types of constructors in C++ with examples.

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MATHEMATICS

THIRD YEAR

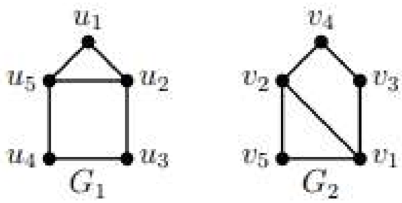
GRAPH THEORY

Time: 3 Hours **Maximum Marks: 75**

PART - A **(5x5 = 25 Marks)**

Answer any FIVE questions.

1. When do you say two graphs are isometric? Check whether the given graphs are isometric



2. Prove that for any positive integer n, a tree with n vertices has n-1 edges.
3. State and prove handshaking theorem
4. Show that an edge e(u;v) is not a cut-edge of G if and only if e belongs to a cycle in G.
5. Prove that Every hamiltonian graph is 2-connected.
6. For any graph G, $\chi(G) \leq 1+\max\delta(G')$ where the summation is taken over all induced subgraphs G'of G.
7. If G is uniquely n-colourable, then $\delta(G) \geq n - 1$.
8. Prove that in a digraph D, sum of all the indegrees of all the vertices is equal to the sum of their out degrees, each sum being equal to the number of arcs in D.

PART - B **(5x10 = 50 Marks)**

Answer any FIVE questions.

9. a) Let G1 be a (p1, q1) and G2 a (p2, q2) graph. Prove the following

   1. G1 ∪ G2 is a (p1 + p2, q1 + q2) graph.

   2. G1 + G2 is a (p1 + p2, q1 + q2 + p1p2) graph.

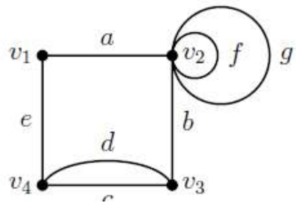
   3. G1 × G2is a (p1p2, q1p2 + q2p1) graph.

   b) prove that A graph G with p points and $\delta \geq \frac{p-1}{2}2$ is connected.

10. a) Prove that with usual notations f or any graph G ,$\kappa \leq \lambda \leq \delta$

    b) Prove that A graph G with p points and $\delta \geq \frac{p-1}{2}2$ is connected.

11. a) For the graph G find its adjacency matrix and incidence matrix.



b) Prove that a vertex v of a connected graph is a cut-vertex if and only if there exist vertices x and y ($\neq$ v) such that every (x; y)-path contains v.

12. Let G be a (p, q) graph. Prove that the following statements are equivalent.

    (1) G is a tree.

    (2) every two points of G are joined by a unique path.

    (3) G is connected and $p = q + 1$

    (4) G is acyclic and $p = q + 1$

13. a) Prove that Every connected graph has a spanning tree. Also prove that for G be a (p, q) connected graph. Then $q \geq p - 1$

    b) Let T be a spanning tree of a connected graph G. Let x = uv be an edge of G not in T. Then prove that T + x contains a unique cycle.

14. a) Prove that Every tree has a centre consisting of either one point or two adjacent points.

    b)Let G be any graph. prove that the following statements are equivalent.

    1. G is 2-colourable.
    2. G is bipartite.

15. a)If G is k-critical, then prove that $\delta(G) \geq k - 1$ and Every k-chromatic graph has at least k vertices of degree at least $k - 1$.

    b) For any graph G, prove that $\chi \leq \Delta + 1$.

16. a) If two digraphs are isomorphic then prove that the corresponding points have the same degree pair.

    b) Prove that a weak digraph D is an Eulerian iff every point of D has equal in-degree and out-degree.